CALCULATION OF THE EXPLOSION OF HEXOGEN IN ALUMINUM

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A calculation is carried out for a spherical explosive charge in elastic–plastic materials with different yield points. Velocity and stress profiles are shown for several instants of time. The zone of increase of internal energy of the material, due to plastic flow, is shown. The values of the energy in the cavity and also the kinetic and plastic energies of the medium are calculated. Comparison with an experiment carried out with an explosion in aluminum [1] showed satisfactory agreement between the calculated and experimental velocity profile at the free surface.

1. Introduction

Experimental data on the characteristics of an explosion in a solid body are incomplete. Velocity profiles, produced by the motion of the free surface remote from the explosion in the elastic zone are well known, as also the size of the cavity after opening, i.e., after discharge of the gases. One of the methods of calculation is given in [2-5]. The satisfactory agreement of the calculations obtained in this paper with the experimental data confirms the validity of the method chosen for describing the medium as an elastic–plastic material. The distribution obtained for the plastic zones of the material, the energy balance during the explosion, and the formation of elastic waves are of interest.

Two zones of motion were distinguished for the calculation in this paper:

1) the explosive zone, in which the detonation is propagated followed by the movement of waves through the explosion products. The equation of state of the explosion products is taken in a form similar to the equation of state of an ideal gas, in accordance with [6];

2) the solid-matter zone, which is bounded by the cavity occupied by the explosion products and the spherical free surface. The medium is considered as an ideal elastic–plastic material with constant parameters. It is assumed that plasticity begins with the achievement of the Mises condition, i.e., when the second invariant of the stress deviator reaches a value determined by the yield point. This value was assumed to be constant in the calculation.

The dependence of the pressure on the volume and internal energy is taken in the Mie-Grüneisen form.

2. System of Equations

For nonstationary elastic–plastic motion in the spherical case in Lagrangian variables, the equations of motion, continuity and energy have the form

$$\frac{\rho_0}{V}\frac{\partial u}{\partial t} = \frac{\partial \sigma_r}{\partial r} + 2\frac{\sigma_r - \sigma_\varphi}{r} \tag{2.1}$$

$$\frac{1}{V}\frac{\partial V}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u), \qquad V = \frac{\rho_0}{\rho} \tag{2.2}$$

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$$\rho_0 \frac{\partial E}{\partial t} = -p \frac{\partial V}{\partial t} + V\left(s_r \frac{\partial e_r}{\partial t} + 2s_\varphi \frac{\partial e_\varphi}{\partial t}\right) \tag{2.3}$$

$$\frac{\partial e_r}{\partial t} = \frac{\partial u}{\partial r}, \quad \frac{\partial e_\varphi}{\partial t} = \frac{u}{r}, \quad \sigma_r = -p + s_r, \quad \sigma_\varphi = -p + s_\varphi \tag{2.4}$$

Here u is the velocity, ρ is the density of the medium, ρ_0 is the initial density, p is the pressure, σ_r and σ_φ are the radial and tangential stresses, s_r and s_φ are the components of the stress deviator, E is the internal energy per unit mass, and e_r and e_φ are the components of the deformation tensor.

The stress–deformation relations for an elastic material are used in the form

$$\frac{\partial s_r}{\partial t} = 2\mu \frac{\partial e_r'}{\partial t}, \quad \frac{\partial s_\varphi}{\partial t} = 2\mu \frac{\partial e_\varphi'}{\partial t}, \quad \frac{\partial e_r'}{\partial t} = \frac{\partial e_r}{\partial t} - \frac{1}{3V}\frac{\partial V}{\partial t}, \quad \frac{\partial e_\varphi'}{\partial t} = \frac{\partial e_\varphi}{\partial t} - \frac{1}{3V}\frac{\partial V}{\partial t} \tag{2.5}$$

Here μ is the shear modulus; e'_r and e'_φ are the components of the deformation deviator. These equations are obtained after differentiation of Hooke's law and the use of the continuity equation.

The Mises condition for the start of plastic flow with spherical symmetry has the form

$$(\sigma_r - \sigma_\varphi)^2 = Y_0^2 \tag{2.6}$$

where Y_0 is the yield point for simple extension.

When satisfying this condition, the Prandtl–Reuss equations should be used instead of Eq. (2.5):

$$2\mu \frac{\partial e_r'}{\partial t} = \frac{\partial s_r}{\partial t} + \lambda s_r, \quad 2\mu \frac{\partial e_\varphi'}{\partial t} = \frac{\partial s_\varphi}{\partial t} + \lambda s_\varphi \tag{2.7}$$

The quantity λ is determined from the yield condition by the equations

$$\lambda = 0 \quad (W < 0), \quad \lambda = \frac{3\mu W}{Y_0^2} \quad (W \geqslant 0), \quad W = s_r \frac{\partial e_r'}{\partial t} + 2s_\varphi \frac{\partial e_\varphi'}{\partial t} \tag{2.8}$$

Here W is the rate of energy dissipation during plastic flow (per unit volume).

The equation of state was used in the form (special case of the Mie–Grüneisen equation)

$$p = a\left(\frac{1}{V} - 1\right) + b\left(\frac{1}{V} - 1\right)^2 + c\left(\frac{1}{V} - 1\right)^3 + \Gamma\rho E \tag{2.9}$$

Here a, b, c, and Γ are experimental constants.

In order to calculate the detonation zone of the explosive and the motion of the explosion products, the equation of state suggested in [6] was used, and this was approximated in the following way:

$$\begin{aligned} p &= p_1\left(\frac{\rho}{\rho_0}\right)^{\gamma_1}, \quad E = \frac{p}{\rho(\gamma_1 - 1)} + \frac{p_*(\gamma_1 - \gamma_2)}{\rho_*(\gamma_2 - 1)(\gamma_1 - 1)} \quad (p \geqslant p_*) \\ p &= p_2\left(\frac{\rho}{\rho_0}\right)^{\gamma_2}, \quad E = \frac{p}{\rho(\gamma_2 - 1)} \quad (p < p_*) \end{aligned} \tag{2.10}$$

The values of the constants for hexogen are $p_* = 1714$ bar, $\gamma_1 = 2.81$, $\gamma_2 = 1.26$, $p_1 = 10.6 \cdot 10^4$ bar, $p_2 = 1.1 \cdot 10^4$ bar, and $\rho_0 = 1.5$ g/cm^3.

Equations (2.1)–(2.3) were used to describe the motion of the explosion products, in which the stress deviator constants were assumed to be equal to zero. Equations (2.10) were used instead of Eq. (2.9).

3. The Difference System of Calculation

Equations (2.1)–(2.10) were approximated by a difference system similar to the one used in [2]. If the stressed state after evaluation by the laws of elasticity proved to be such that the left-hand side of Eq. (2.6) is greater than the right-hand side, reduction of the stress deviator was carried out by multiplying s_r and s_φ by the factor n

$$n = \sqrt{2/3} Y_0 (s_r^2 + 2s_\varphi^2)^{-1/2} \tag{3.1}$$

This recalculation corresponds to the application of the complete relations of the theory of plasticity (2.7) and (2.8) [2].

The scheme from [2] was supplemented by the use of a special type of averaging.

After calculating all the parameters in a new layer with respect to time, averaging was carried out as necessary in order to reduce variations. When calculating elastic–plastic problems without averaging, longitudinal oscillations are observed during transition from the zone of elasticity to the zone of plasticity and vice versa, the calculation of the stresses in which are carried out with different formulas.

In this scheme, the velocity was averaged only after its new value had been calculated. Calculation of the specific volume, pressure, and coordinates was effected with averaged velocity values.

Velocity averaging was carried out in each layer with respect to time but not at every countable point, and only at those around which the scaled velocities had different signs to left and right. At the first and last points, averaging was not carried out, for the purpose of improving the calculation of the shock waves. Consequently, averaging was not carried out entirely in the monotonic profile.

Averaging was effected at the point i, if the condition

$$\text{sign}\,(u_i - u_{i-1}) \neq \text{sign}\,(u_{i+1} - u_i) \tag{3.2}$$

was fulfilled.

We write the momentum equation before and after velocity averaging:

$$^1/_2 [(u_{i-1} + u_i)\, m_{i-1/2} + (u_i + u_{i+1})\, m_{i+1/2}] = \langle u_i \rangle (m_{i-1/2} + m_{i+1/2}) \tag{3.3}$$

The quantity $m_{i+1/2}$ is the mass of the medium between the points i and i + 1.

We obtain the equation by which velocity averaging was carried out:

$$\langle u_i \rangle = \frac{u_i}{2} + \frac{u_{i+1} m_{i+1/2} + u_{i-1} m_{i-1/2}}{2 (m_{i+1/2} + m_{i-1/2})} \tag{3.4}$$

Here $\langle u_i \rangle$ is the averaged value of the velocity.

In order to verify the working of the scheme, a calculation of the problem was carried out in the elastic region, for which there is an exact analytic solution: when t = 0, a constant pressure originates in the cavity and is maintained. A satisfactory agreement is obtained between the calculated and exact solution in the region of continuous flow. The shock wave diffuses with an artificial viscosity.

4. Results of the Calculations

In this paper, calculations have been carried out for explosions of hexogen in aluminum Al-00, annealed Do-16, and quenched Dural Dz-16. The constants, defining their properties, are shown in Table 1. These values are taken to be the same as in [6].

In the calculation, the radius of the explosive was r_0 = 3.2 mm. The pitch over the space in the region of the explosive was 0.05 mm. Reflection from the center was processed like reflection from a rigid

TABLE 1*

No. on graphs	Material	ρ_0, g/cm³	c_l, km/sec	ν	σ_s, kbar	a, kbar	b, kbar	c, kbar	μ, kbar	Γ
1	Al — 00	2.71	6.40	0.36	0.44	787	1.175	380	243	1
2	Do — 16	2.78	6.47	0.33	1.57	770	1.343	360	295	1
3	Dz — 16	2.81	6.57	0.33	2.89	803	1.210	400	308	1

*Here c_l is the longitudinal velocity of sound; σ_s is the yield point.

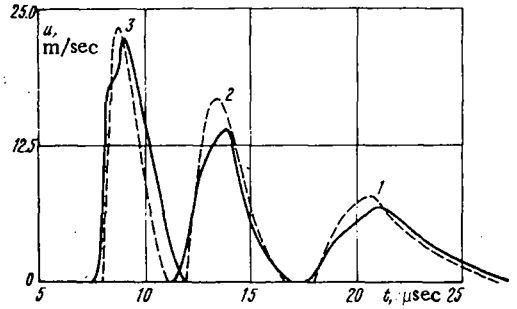

Fig. 1

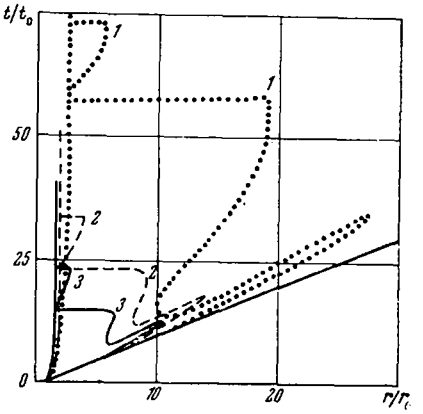

Fig. 2

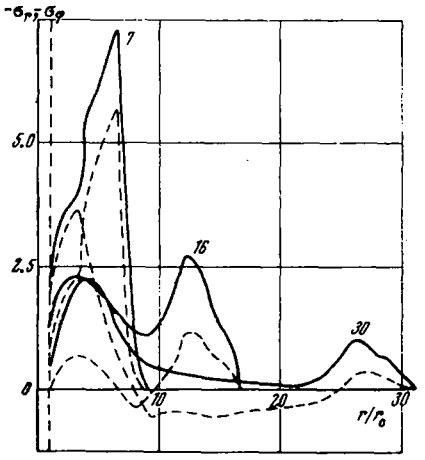

Fig. 3

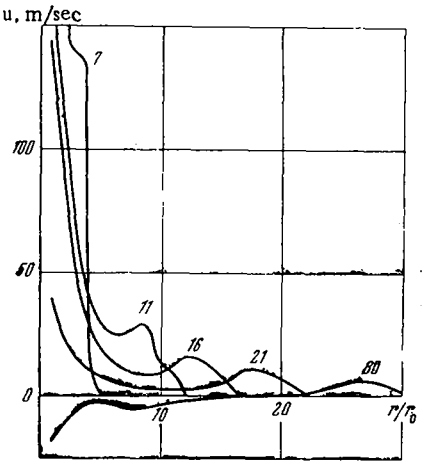

Fig. 4

sphere of radius 0.051 mm.

Calculation of the detonation was carried out by the "spreading" of the energy release at certain points. The rate of travel of the energy release zone was equivalent to an assigned detonation velocity of $7.56 \cdot 10^5$ cm/sec. The calorific value of the explosive was assumed equal to 1345 kcal/kg [6].

As a result of the calculation, it was found that at the instant of emergence at the boundary of the explosive, the pressure profile of the computed zone coincides satisfactorily with the self-similar profile of a spherical detonation.

For comparison with experiment, the explosion in aluminum was computed in the presence of a free surface at distances of 11.8, 7.9, and 5.4 cm from the center of the explosion for materials 1, 2, and 3 respectively, in accordance with the data of the table.

Figure 1 shows the velocity of the free surface as a function of time for the three media, differing only in yield point (see table). The experimental velocity profiles [1], obtained for the very same values of the parameters of the problem as in the calculation, are shown dashed. The numbers denote the number of the material in the table. The agreement is satisfactory with respect to amplitude and wavelength.

Calculations were carried out in which the free surface was located at a distance from the explosion. Figures 2 to 6 represent the results of the calculations in which the shock wave had not yet reached the free surface.

Figure 2 shows the regions of plasticity in the time–distance diagram for the three materials. An elastic wave is moving in front with constant velocity. Behind it, also with constant velocity, a plastic wave is moving. The width of this first region of plasticity is determined by the appropriate artificial viscosity, i.e., in fact its width is equal to zero as, following it, a rarefaction wave is propagating. When the stress is reduced to zero in the plasticity wave, this region disappears at the times $34t_0$, $18t_0$, and $12t_0$, respectively, for materials 1, 2, and 3.

The second region of plasticity starts after the rarefaction wave, close to the cavity. Transition to elastic stresses occurs later than in the first region; stress relief begins at the boundary of the cavity and at a high rate, exceeding the velocity of sound by a factor of ten, encompasses the entire region. At later times, with reversal of the motion of the cavity, yet another region of plasticity adjacent to the cavity originates in all three materials. It also ends in rapid stress relief.

Figure 3 shows the values of the stress σ_r (continuous lines) and σ_φ (dashed lines) in kbar at the times $7t_0$, $16t_0$, and $30t_0$ for a calculation of an explosion in unconfined annealed Dural Do-16 (material 2). The quantity $t_0 = r_0/c_l$ is equal to the transit time of a longitudinal wave to a distance equal to the charge radius. The distances r here and in Figs. 4 and 5 are Lagrangian coordinates of particles equal to their initial distance from the center of the explosion.

The first pair of profiles corresponds to the time $7t_0$ when plastic flow occurs in the region from the front and up to the cavity, in accordance with Fig. 2 (material 2).

The next pair of profiles is given at the instant $16t_0$. Here, there are two regions of plasticity: one close to the wave front and the other close to the cavity.

At the time $t = 30t_0$, plastic flow occurs only close to the cavity, around which movement at the center of the explosion occurs. Here $|\sigma_\varphi| > |\sigma_r|$. At earlier times, the inequality sign is opposite.

Figure 4 shows the velocity profiles for the calculation of an explosion in material 2. The figures near the curves denote the magnitude of the ratio t/t_0. When $t = 30t_0$, there exists a large region of reverse motion right up to $r = 21r_0$. However, only close to the cavity up to $r = 3.5r_0$ does plastic flow occur.

Qualitatively similar relations are obtained for materials 1 and 3.

Figures 5 and 6 show the results of an energy calculation of a different form.

In Fig. 5, the continuous lines represent the change with time of the magnitude of the total plastic energy E_p relative to the explosion energy E_0. This part of the explosion energy is expended on heating up the medium. The figures near the curves denote the number of the material. We note the weak dependence of the final value of this energy on the yield point of the material (48.5% in Al-00; 44.5% in Do-16; and 42.0% in Dz-16). In the more durable material, the plastic energy is confined to a small region in the vicinity of the explosion, as only here are there sufficiently large stresses. Heating of the medium in the more durable material is considerably greater. This is shown in Fig. 5 by the dashed lines. Here, the final increment of the specific thermal internal energy ΔE_T in cal/g for the three materials (1, 2, and 3) is shown as a function of the relative distance r/r_0.

The total energy balance at different instants is shown in Fig. 6. The continuous line shows the calculation for an explosion in Dz-16, the dashed line for Do-16, and the dots for Al-00.

The distance from the axis of abscissa to the lower curve is proportional to the energy in the cavity E_c. This energy in the more durable material is greater, as the cavity in it is expanding less. Ultimately, 22.5, 31.5, and 36% of the total explosion energy remains in the cavity, for materials 1, 2, and 3 respectively.

The distance from this curve to the next one is proportional to the plastic energy E_p.

The magnitude of the kinetic energy E_k has a maximum at the instant when the elastic wave passes to a distance of 10, 4, and 3 charge radii in materials 1, 2, and 3 respectively. After plastic flow ceases,

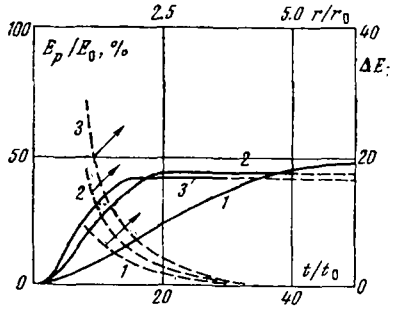

Fig. 5

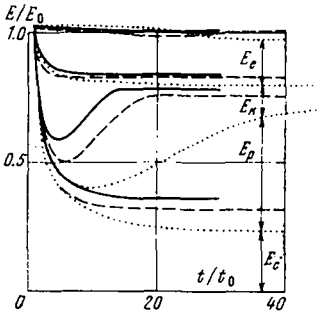

Fig. 6

8, 7, and 6% respectively of the total explosion energy remains as kinetic energy.

The magnitude of the elastic deformation energy E_e is plotted above the kinetic energy curve.

The accuracy of the calculations can be judged by the upper curve. The total energy imbalance does not exceed 3%. Variations occur because of the elastic energy.

The calculations carried out showed the relatively weak dependence of the final energy distribution on the yield point of the material.

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LITERATURE CITED

1. V. N. Rodionov and A. P. Sukhotin, "Parameters of elastic waves, emitted as a result of a spherical explosion in metals with different durability," Fiz. Goreniya i Vzryva, 7, No. 1 (1971).
2. M. L. Wilkins, "Calculation of elastic–plastic flows," in: Numerical Methods in Hydrodynamics [Russian translation]. Mir, Moscow (1967).
3. R. T. Allen and R. E. Duff, "Effect of material properties on cavity size from an underground nuclear explosion," Nuclear Appl., 6, No. 6 (1969).
4. L. V. Al'tshuler, A. V. Balabanov, V. A. Batalov, N. A. Gerashchenko, V. A. Rodionov, V. A. Svidinskii, and D. M. Tarasov, "A contained explosion in liquid and elastic–plastic media," Dokl. Akad. Nauk SSSR, 193, No. 6 (1970).
5. V. A. Batalov and V. A. Svidinskii, "Investigation of the effect of the parameters of the medium on the final dimensions of the cavity in the case of a powerful contained explosion," Izv. Akad. Nauk SSSR, Fizika Zemli, No. 12 (1971).
6. N. M. Kuznetsov and K. K. Shvedov, "Equation of state of the detonation products of hexogen," Goreniya i Vzryva, No. 4 (1966).